

1-3 Radicals

Definition

 n th root

$$\sqrt[n]{b} = a \text{ means } b = a^n$$

- if $n \geq 2$ and even then a and b must be greater than or equal to 0.
- if $n \geq 3$ and odd, then a and b can be any real number.

In $\sqrt[n]{b}$:The symbol $\sqrt{\quad}$ is called the radical n is called the index b is called the radicand

if there is no index, it is 2

Evaluate

$$\sqrt[2]{9} = 3$$

Handwritten work: $\sqrt[2]{9} = 3$ with a green box around the 3. Below the 9 are two 3s with a vertical line between them, circled in black.

$$\sqrt{49} = 7$$

Handwritten work: $\sqrt{49} = 7$ with a green box around the 7. Below the 49 are two 7s with a vertical line between them, circled in black.

$$\sqrt[4]{16} = 2$$

Handwritten work: $\sqrt[4]{16} = 2$ with a green box around the 2. Below the 16 are two 4s with a vertical line between them, and below the 4s are four 2s with vertical lines between them, circled in black.

$$\sqrt[3]{64} = 4$$

Handwritten work: $\sqrt[3]{64} = 4$ with a green box around the 4. Below the 64 are two 8s with a vertical line between them, and below the 8s are four 2s with vertical lines between them, circled in blue.


$$\sqrt[3]{-8} = -2$$

Handwritten work: $\sqrt[3]{-8} = -2$ with a green box around the -2. Below the -8 are two 4s with a vertical line between them, and below the 4s are two -2s with a vertical line between them, circled in blue.

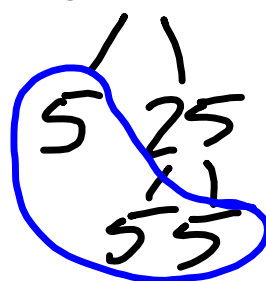
$$\sqrt[4]{81} = 3$$

Handwritten work: $\sqrt[4]{81} = 3$ with a green box around the 3. Below the 81 are two 9s with a vertical line between them, and below the 9s are four 3s with vertical lines between them, circled in blue.

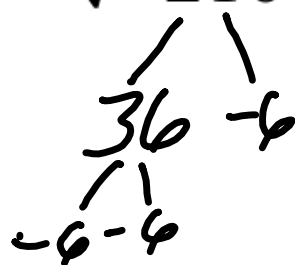
You try

$$\sqrt{121} = 11$$


A handwritten prime factorization of 121. The number 121 is written with a square root symbol over it. Below the 121, there are two diagonal lines forming a V-shape, with the number 11 written under each line.

$$\sqrt[3]{125} = 5$$


A handwritten prime factorization of 125. The number 125 is written with a cube root symbol over it. Below the 125, there are two diagonal lines forming a V-shape, with the number 5 written under the left line and 25 written under the right line. Below 25, there are two diagonal lines forming a V-shape, with the number 5 written under each line. The entire factorization (5, 25, 5, 5) is circled in blue.

$$\sqrt[3]{-216} = -6$$


A handwritten prime factorization of -216. The number -216 is written with a cube root symbol over it. Below the -216, there are two diagonal lines forming a V-shape, with the number 36 written under the left line and -6 written under the right line. Below 36, there are two diagonal lines forming a V-shape, with the number -6 written under each line.

$$\sqrt[5]{32} = 2$$

Simplifying

If $n \geq 2$ is a positive integer and a is a real number, then

$$\sqrt[n]{a^n} = a \quad \text{if } n \geq 3 \text{ is odd}$$

$$\sqrt[n]{a^n} = |a| \quad \text{if } \underline{n \geq 2} \text{ is even}$$

Reduce

$$\sqrt{x^2} = x$$

$$\sqrt[5]{x^5} = x$$

You try

$$\sqrt[3]{x^3} = x$$

$$\sqrt[6]{z^6} = |z|$$

Simplify

$$\sqrt{18} = 3\sqrt{2}$$

$$\begin{array}{c} \diagup \quad \diagdown \\ 9 \quad 2 \end{array}$$

$$\begin{array}{c} \diagup \quad \diagdown \\ \textcircled{3} \quad \textcircled{3} \end{array}$$

$$\sqrt{9} \cdot \sqrt{2}$$

Simplify

(remember $\sqrt{x^2} = |x|$)

$$5\sqrt[3]{24}$$

$$\begin{array}{c} \diagup \quad \diagdown \\ 12 \cdot 2 \\ \diagup \quad \diagdown \\ 6 \quad 2 \\ \diagup \quad \diagdown \\ \color{red}{3} \quad 2 \end{array}$$

$$5 \cdot 2\sqrt{3}$$

$$\color{green}{10\sqrt{3}}$$

$$\color{green}{\sqrt[4]{20}}$$

$$\begin{array}{c} \diagup \quad \diagdown \\ 10 \quad 2 \\ \diagup \quad \diagdown \\ 5 \quad 2 \end{array}$$

$$\sqrt{128x^2} = \cancel{8|x|}$$

$$\begin{array}{c} \diagup \quad \diagdown \\ 2 \quad 64 \\ \diagup \quad \diagdown \\ \color{blue}{8} \quad 8 \end{array}$$

$$\color{green}{8|x|\sqrt{2}}$$

You try

$$\sqrt{48} = 4\sqrt{3}$$

$\begin{array}{c} \diagup \quad \diagdown \\ 12 \quad 4 \\ \diagdown \quad \diagup \\ 3 \quad 4 \end{array}$

$$4\sqrt[3]{54} = 4 \cdot 3\sqrt{2}$$

$\begin{array}{c} \diagup \quad \diagdown \\ 27 \quad 2 \\ \diagdown \quad \diagup \\ 9 \quad 3 \\ \diagdown \quad \diagup \\ 3 \quad 3 \end{array}$

$$\sqrt{200a^2} = 10|a|\sqrt{2}$$

$\begin{array}{c} \diagup \quad \diagdown \\ 100 \quad 2 \\ \diagdown \quad \diagup \\ 10 \quad 10 \end{array}$

$$\sqrt[4]{40}$$

$\begin{array}{c} \diagup \quad \diagdown \\ 20 \quad 2 \\ \diagdown \quad \diagup \\ 5 \quad 4 \\ \diagdown \quad \diagup \\ 2 \quad 2 \end{array}$

Simplify

$$\sqrt{12p^2q} \quad 2|P \quad |\sqrt{3q}$$

Handwritten work for simplifying $\sqrt{12p^2q}$:

Factorization of 12: $12 = 3 \times 4$

Factorization of p^2 : $p^2 = p \times p$

Final simplified form: $2p\sqrt{3q}$ (The $2p$ is circled in blue in the original image)

Remember that

$$\sqrt[n]{a^n} = a \quad \text{if } n \geq 3 \text{ is odd}$$

$$\sqrt[n]{a^n} = |a| \quad \text{if } n \geq 2 \text{ is even}$$

For example

$$\sqrt{x^2} = |x| \quad \sqrt[3]{x^3} = x \quad \sqrt[4]{x^4} = |x| \quad \text{and so on}$$

But to make our life easier some instructions will say "Assume all variables are greater than or equal to zero." In which case:

$$\sqrt{x^2} = x \quad \sqrt[3]{x^3} = x \quad \sqrt[4]{x^4} = x \quad \text{and so on}$$

SO READ YOUR INSTRUCTIONS!!!

Reduce Assuming all variables are greater than or equal to zero.

(You can either do these using rational exponents or not.)

$$\sqrt[2]{x^{6/2}} = x^3$$

$$x \cdot x \cdot x \cdot x \cdot x \cdot x$$

$$\sqrt[3]{x^{12/3}} = x^4$$

$$x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$$

$$\sqrt[3]{x^{14}}$$

$$\sqrt[3]{x^{12}} \cdot \sqrt[3]{x^2}$$

$$x^4 \sqrt[3]{x^2}$$

You try

$$\sqrt{48}$$

$$4\sqrt[3]{54}$$

$$\sqrt{200a^2}$$

$$\sqrt[4]{40}$$

Reduce Assuming all variables are greater than or equal to zero.

$$\sqrt{20x^{10}}$$

You try

$$\sqrt{75a^6} = 5a^3/\sqrt{3}$$

Handwritten prime factorization of 75:

$$\begin{array}{c} \sqrt{} \\ / \quad \backslash \\ 25 \quad 3 \\ / \quad \backslash \\ \textcircled{5} \quad 5 \end{array}$$

Simplify Assuming all variables are greater than or equal to zero.

$$\sqrt{80a^3}$$

$$\begin{array}{l} \textcircled{3} \sqrt{27m^4n^{14}} \\ \begin{array}{l} 1 \ 1 \\ 9 \ 3 \\ 4 \ 1 \\ 3 \ 3 \end{array} \quad \begin{array}{l} m \ m \ m \ m \\ n^{\frac{12}{3}} \ n^2 \end{array} \\ \hline 3m^4 \sqrt{mn^2} \end{array}$$

You Try

$$\sqrt[3]{128x^6y^{10}}$$

$$\sqrt[4]{16a^5b^{11}}$$

26)

$$\frac{4n^3}{-n \cdot 3n^2}$$
$$\frac{4n^3 \cancel{n^1}}{3 \cancel{n^1}}$$
$$\frac{4n^3}{3}$$

27) $-\frac{x^4 y^{-1} x^{-2}}{3 y^{-4}}$

$-\frac{x^{4-2} y^{4-1}}{3 x^0 y^1} =$

$\frac{-x^2 y^3}{3 y}$

$\frac{-x^2 y^3}{3 y}$

$$23) \quad -\frac{4x^2y^3}{3x^{-3}} = \frac{-4x^{\textcircled{2}}y^{\textcircled{3}}x^{\textcircled{3}}}{3} \\ = \frac{-4x^5y^3}{3}$$

$$\begin{aligned}
 20) \quad & (2x^4y^3 - x^4y^3)^2 \\
 & (2 \cdot -1 \overset{-4}{x} \cdot \overset{4}{x} y^{\overset{3}{y}} y^{\overset{3}{y}})^2 \\
 & (-2x^0y^6)^2 \\
 & (-2)^2 y^{12} \\
 & 4y^{12} \text{ or } 2^2 y^{12}
 \end{aligned}$$