1-3 Radicals

Definition
nth root

$$\sqrt[n]{b}. = a \quad \text{means} \quad b = a^n$$

- if $n \geq 2$ and even then a and b must be greater than or equal to 0.
- if $n \geq 3$ and odd, then a and b can be any real number.

In $\sqrt[n]{b}$:

The symbol $\sqrt{}$ is called the radical

n is called the index

b is called the radicand

if there is no index, it is 2

Evaluate

$\sqrt[2]{9} = \boxed{3}$

$3 \; 3$

$\sqrt{49} = \boxed{7}$

$7 \cdot 7$

$\sqrt[4]{16} = \boxed{2}$

$4 \; 4$

$2 \; 2 \; 2 \; 2$

$\sqrt[3]{64} = 2 \cdot 2 = \boxed{4}$

$8 \; 8$

$4 \; 2 \; 2 \; 4$

$2 \; 2 \qquad 2 \; 2$

$\sqrt[3]{-8} = \boxed{-2}$

$4 \; -2$

$-2 \; -2$

$\sqrt[4]{81} = \boxed{3}$

$9 \; 9$

$3 \; 3 \; 3 \; 3$

You try

$\sqrt{121} = 11$

11 11

$\sqrt[3]{125} = 5$

5 25

5 5

$\sqrt[3]{-216} = -6$

36 -6

-6 -6

$\sqrt[5]{32} = 2$

Simplifying

If n≥2 is a positive integer and a is a real number, then

$$\sqrt[n]{a^n} = a \quad \text{if } n \geq 3 \text{ is odd}$$

$$\sqrt[n]{a^n} = \boxed{|a|} \quad \text{if } n \geq 2 \text{ is even}$$

Reduce

$$\sqrt{x^2} = |x| \qquad \sqrt[5]{x^5} = x$$

xx $\qquad$ $xxxxx$

You try

$\sqrt[3]{x^3} = x$

$\sqrt[6]{z^6} = |z|$

Simplify

$$\sqrt{18} = 3\sqrt{2}$$

$$18 = 9 \cdot 2$$

$$9 = 3 \cdot 3$$

$$\sqrt{9} \cdot \sqrt{2}$$

Simplify

(remember $\sqrt{x^2} = |x|$)

$5\sqrt[3]{24}$ $5 \cdot 2\sqrt{3}$

24: 12 · 2; 12: 6 · 2; 6: 3 · 2

$10\sqrt{3}$

$\sqrt{128x^2} = 8|x|$

128: 2 · 64; 64: 8 · 8

$8|x|\sqrt{2}$

$\sqrt[4]{20}$

20: 10 · 2; 10: 5 · 2

You try

$\sqrt{48} = 4\sqrt{3}$

12 4

3 4

$4\sqrt[3]{54} = 4 \cdot 3\sqrt{2}$

$12\sqrt{2}$

27 2

9 3

3 3

$\sqrt{200a^2} = 10|a|\sqrt{2}$

100 2

10 10

$\sqrt[4]{40}$

20 2

5 4

2 2

Simplify

$\sqrt{12p^2q}$ $\quad 2|p|\sqrt{3q}$

3 4

2 2

Remember that

$$\sqrt[n]{a^n} = a \quad \text{if } n \geq 3 \text{ is odd}$$

$$\sqrt[n]{a^n} = |a| \quad \text{if } n \geq 2 \text{ is even}$$

For example

$$\sqrt{x^2} = |x| \qquad \sqrt[3]{x^3} = x \qquad \sqrt[4]{x^4} = |x|$$

and so on

But to make our life easier some instructions will say "Assume all variables are greater then or equal to zero." In which case:

$$\sqrt{x^2} = x \qquad \sqrt[3]{x^3} = x \qquad \sqrt[4]{x^4} = x$$

and so on

SO READ YOUR INSTRUCTIONS!!!

Reduce Assuming all variables are greater then or equal to zero.

(You can either do these using rational exponents or not.)

$$\sqrt[2]{x^{6/2}} = x^3$$

xx xx xx

$$\sqrt[3]{x^{12/3}} = x^4$$

xxx xxx xxx xxx

$$\sqrt[3]{x^{14}}$$

$$\sqrt[3]{x^{12}} \cdot \sqrt[3]{x^2}$$

$$x^4\sqrt[3]{x^2}$$

You try

$$\sqrt{48}$$

$$4\sqrt[3]{54}$$

$$\sqrt{200a^2}$$

$$\sqrt[4]{40}$$

Reduce Assuming all variables are greater then or equal to zero.

$$\sqrt{20x^{10}}$$

You try

$$a\sqrt{75a^{6/2}} = 5|a^3|\sqrt{3}$$

25 3

5 5

Simplify Assuming all variables are greater then or equal to zero.

$$\sqrt{80a^3} \qquad \sqrt[3]{27m^4n^{14}}$$

9 3

33

mmmmm

$n^{\frac{12}{3}}$ n^2

$3mn^4\sqrt{mn^2}$

You Try

$$\sqrt[3]{128x^6y^{10}}$$

$$\sqrt[4]{16a^5b^{11}}$$

26) $\dfrac{\cancel{7}4n^3}{\cancel{7}n \cdot 3n^{-1}}$

$$\frac{4n^3\cancel{n^1}}{3\cancel{n}}$$

$$\boxed{\frac{4n^3}{3}}$$

27) $-\dfrac{x^4 y^{-1} x^{-2}}{3y^{-4}}$

$-\dfrac{x^{4-2} y^{4-1}}{3x^2 y^1} = \dfrac{-\cancel{x}\cancel{x}xx\cancel{y}yyy}{3\cancel{x}\cancel{x}\cancel{y}}$

$\dfrac{-x^2 y^3}{3}$

23) $-\dfrac{4x^2y^3}{3x^{-3}} = \dfrac{-4x^2y^3x^3}{3}$

$= \dfrac{-4x^5y^3}{3}$

20) $(2x^{-4}y^3 \cdot -x^4y^3)^2$

$(2 \cdot -1\, x^{-4} \cdot x^4\, y^3 \cdot y^3)^2$

$(-2\cancel{x^0}\, y^6)^2$

$(-2)^2\, y^{12}$

$4y^{12}$ or 2^2y^{12}