10-3

Radian Measure, Arc Length, and Sectors

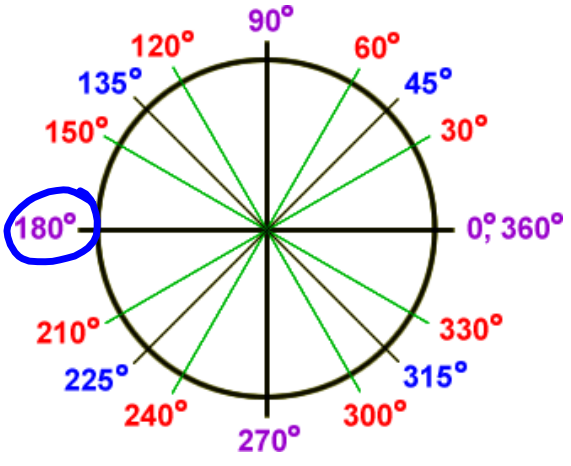
I can convert between degrees and radians

I can find exact trig values

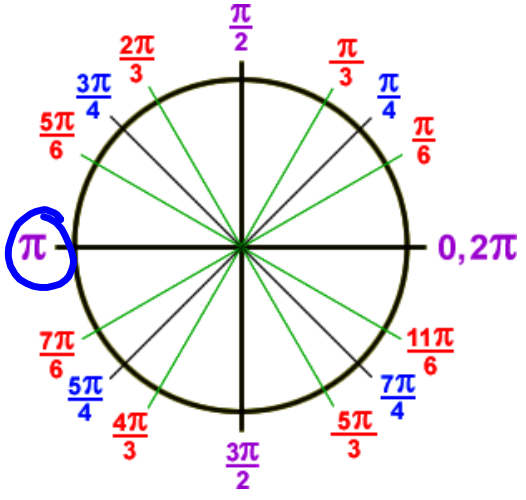
I can find arc lengths

I can find the area of sectors

Degrees



Radians



Converting between degrees and radians:

$$x^\circ \bullet \frac{\pi}{180^\circ} = radians$$

$$xradians \bullet \frac{180^\circ}{\pi} = \text{deg}rees$$

OR

$$\frac{rad}{\pi} = \frac{\text{deg}}{180^\circ}$$

Example:

Convert degrees to radians:

a) $90^\circ \cdot \frac{\pi}{180} = \frac{1\pi}{2}$ b) $135^\circ \cdot \frac{\pi}{180} = \frac{3\pi}{4}$

Convert radians to degrees:

c) $-\frac{3\pi}{4} \cdot \frac{180}{\pi} = -135$ d) $\frac{16\pi}{9} \cdot \frac{\cancel{180}^{20}}{\pi} = 320^\circ$

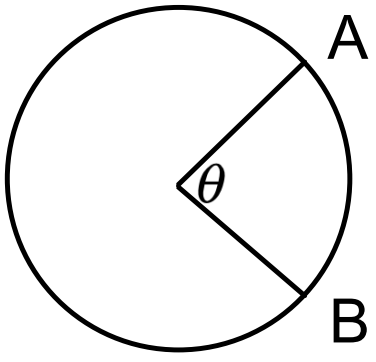
Recall:

Circumference of a circle

$$C = 2\pi r$$

Arc Length:

A proportion of the circumference of the circle. You can use the measure of the arc (in degrees) to find its length (in linear units.)



Degrees

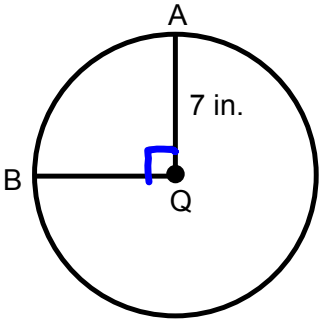
$$arclength = \left(\frac{\overset{\frown}{AB}}{360^o} \right) 2\pi r$$

Radians

$$arclength = \theta r$$

Example:

Find the length of $\overset{\frown}{AB}$



$AL = \frac{\theta}{360} \cdot 2\pi r$

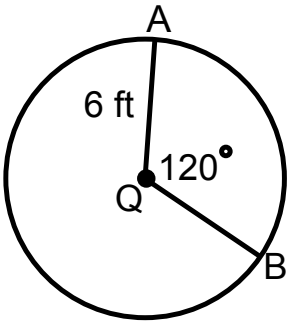
$\rightarrow = \frac{90}{360} \cdot 2\pi(7)$

$= 10.995$

$\rightarrow = 11 \text{ in}$

You Try:

Find the length of $\overset{\frown}{AB}$

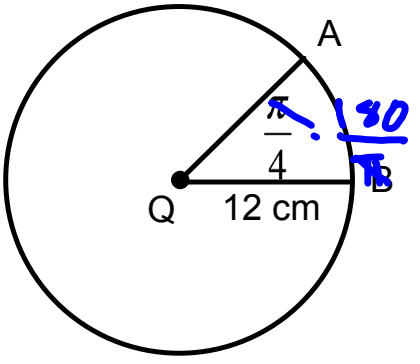


$$AL = \frac{\theta}{360} \cdot 2\pi r$$

$$AL = \frac{120}{360} \cdot 2\pi(6) = 12.56$$

Example:

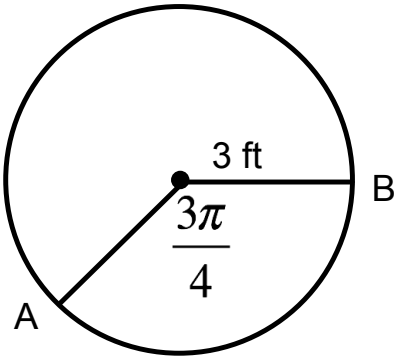
Find the length of $\overset{\frown}{AB}$



$\frac{45}{360} \cdot \pi R(2)$ ($R = 12$) $= 9.424$ cm

You try:

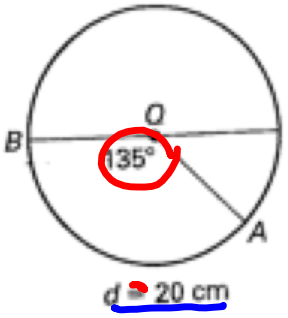
Find the length of $\overset{\frown}{AB}$



Example:

Find the indicated measure of each:

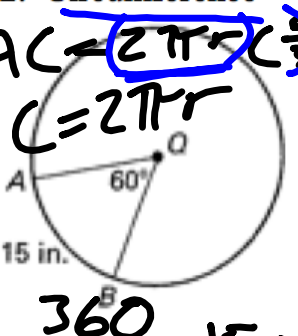
1. Length of $\overset{\frown}{AB}$



$\frac{135}{360} = 47.123$

$\frac{135}{360} \cdot 2\pi(10) = 23.56$

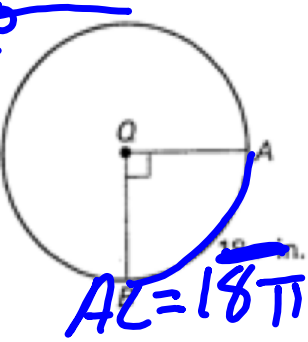
2. Circumference



$\frac{360}{60} \cdot 15 = C$

$90 = C$

3. Radius



$AC = 18\pi$

$\frac{18\pi}{2\pi} = \frac{2\pi r}{2\pi} \frac{90}{360}$

$9 = r \cdot \frac{90}{360} \cdot \frac{360}{90}$

$36 = r$

Recall:

Area of a circle

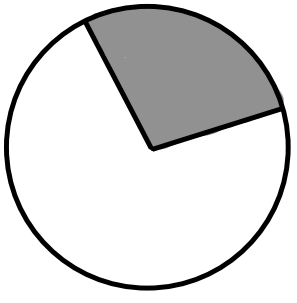
$$A = \pi r^2$$

Sectors:

A sector of a circle is the region (area) bounded by two radii of the circle and their intercept arc.

$AL = \frac{\theta}{360} C$

$SA = \frac{\theta}{360} A$

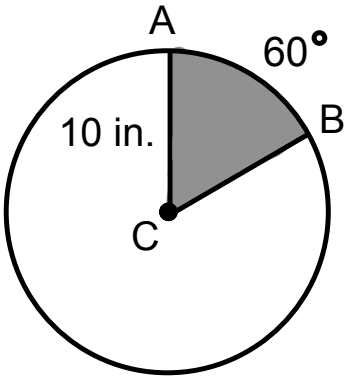


Degrees

$$Sectorarea = \left(\frac{\overset{\frown}{AB}}{360^o} \right) \pi r^2$$

Example:

Find the area of the sector:



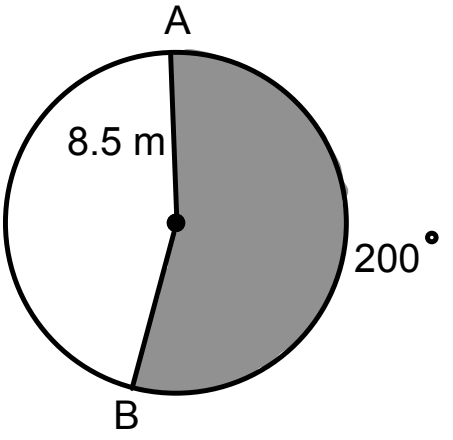
$$SA = \frac{\theta}{360}\pi r^2$$

$$= \frac{60}{360}\pi(10)^2$$

$$= 52.4 \text{ in}^2$$

Example:

Find the shaded area:



$$SA = \frac{\theta}{360} \cdot \pi r^2$$

$$= \frac{200}{360} \pi (8.5)^2$$

$$= 126.1 \text{ m}^2$$

You try:

Find the area of the shaded region: